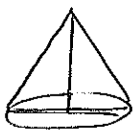
AP Calculus AB
Related Rates Worksheet #2

Name Key

Sand is pouring from a pipe at a rate of 16 cubic feet per second. The falling sand forms a conical pile on the ground. The altitude of the pile is always ¼ the diameter of the base. What is the rate of change of the altitude at the instant that the altitude is 4 feet?



$h = \frac{1}{4} d$

$h = \frac{1}{4}(2r)$

$h = \frac{r}{2}$

$r = 2h$

$\frac{dh}{dt} = ?$

$h = 4$ ft.

$\frac{dV}{dt} = 16$ ft³/sec

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}(2h)^2 h$

$V = \frac{\pi}{3} 4h^2 \cdot h$

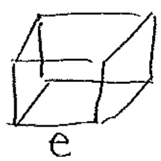
$V = \frac{4\pi}{3} h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$(16) = 4\pi(4)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{4\pi}$ ft/sec

2. The edge of a cube is expanding at a rate of 2.5 centimeters per second. How fast is the volume changing when the length of an edge is 15 centimeters? How fast is the surface area changing at the same instant?



$\frac{de}{dt} = 2.5$ cm/sec

$\frac{dV}{dt} = ?$

$e = 15$ cm

$V = e^3$

$\frac{dV}{dt} = 3e^2 \frac{de}{dt}$

$\frac{dV}{dt} = 3(15)^2(2.5)$

$\frac{dV}{dt} = 1687.5$ cm³/sec

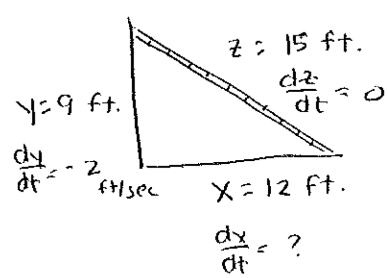
$A = 6e^2$

$\frac{dA}{dt} = 12e \frac{de}{dt}$

$\frac{dA}{dt} = 12(15)(2.5)$

$\frac{dA}{dt} = 450$ cm²/sec

3. A 15-foot ladder leans against a vertical wall. If the top of the ladder slides down the wall at a rate of 2 feet per second, how fast is the bottom of the ladder moving when it is 12 feet from the wall?



$x^2 + y^2 = z^2$

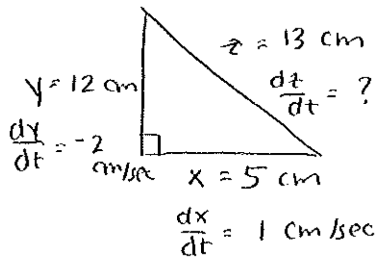
$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = \cancel{2}z \frac{dz}{dt}$

$(12)\frac{dx}{dt} + (9)(-2) = (15)(0)$

$12 \frac{dx}{dt} = 18$

$\frac{dx}{dt} = 1.5$ ft/sec

4. At a given instant, the legs of a right triangle are 5 cm and 12 cm long. If the short leg is increasing at a rate of 1 cm per second and the long leg is decreasing at a rate if 2 cm per second, how fast is the hypotenuse changing?



$$x^2 + y^2 = z^2$$

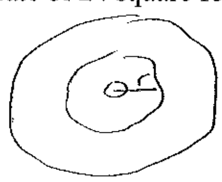
$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$

$$(5)(1) + (12)(-2) = (13)\frac{dz}{dt}$$

$$-19 = 13\frac{dz}{dt}$$

$$\frac{dz}{dt} = -1.462 \text{ cm/sec}$$

5. A stone is dropped into a still pond and it produces circular ripples. If the radius of one of the ripples increases at a rate of 2 feet per second, how fast is the enclosed area changing when the radius is 12 feet? If the area is changing at a rate of 24 square feet per second, how fast is the radius changing when the area is 130 square feet?



$\frac{dr}{dt} = 2$ ft/sec

$\frac{dA}{dt} = ?$

$r = 12$ ft.

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(12)(2)$$

$$\frac{dA}{dt} = 48\pi \text{ ft}^2\text{/sec.}$$

$\frac{dA}{dt} = 24$ ft²/sec

$\frac{dr}{dt} = ?$

$A = 130$ ft²

$$130 = \pi r^2$$

$$r = \sqrt{\frac{130}{\pi}}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

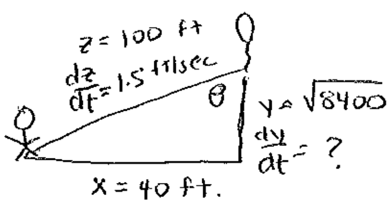
$$(24) = 2\pi\left(\sqrt{\frac{130}{\pi}}\right)\frac{dr}{dt}$$

$$\frac{dr}{dt} = .594 \text{ ft/sec}$$

6. A balloon is rising vertically. The horizontal distance between the balloon and an observer on the ground is 40 feet. The distance between the observer and the rising balloon is changing at a rate of 1.5 feet per second.

a) At the instant when the distance between the observer and the balloon is 100 feet, how fast is the vertical distance changing?

b) At the instant when the distance between the observer and the balloon is 100 feet, how fast is the angle between the hypotenuse and the vertical distance changing?



a) $x^2 + y^2 = z^2$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$(40)(0) + (\sqrt{8400})\left(\frac{dy}{dt}\right) = (100)(1.5)$$

$$\frac{dy}{dt} = 1.637 \text{ ft/sec}$$

b) $\sin\theta = \frac{x}{z}$

$$\cos\theta\frac{d\theta}{dt} = \frac{\left(\frac{dx}{dt}\right)(z) - (x)\left(\frac{dz}{dt}\right)}{z^2}$$

$$\left(\frac{\sqrt{8400}}{100}\right)\frac{d\theta}{dt} = \frac{(0)(100) - (40)(1.5)}{(100)^2}$$

$$\frac{d\theta}{dt} = .007 \text{ rad/sec}$$